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A NOTE ON THE USE OF LOCAL MIN AND MAX OPERATIONS IN DIGITAL PICTURE PROCESSING.

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ABSTRACT

"Shrinking" and "expanding" operations on two-valued digital pictures are useful for noise removal, as well as for detecting "dense" regions and elongated parts of objects. For grayscale pictures, the analogs of shrinking and expanding are local MIN and MAX operations. These operations commute with thresholding; thus if they are applied to a grayscale picture, followed by thresholding, the result is the same as if the picture were first thresholded and shrinking and expanding were then performed. Applying MIN and MAX operations prior to thresholding thus makes it possible to defer the choice of a threshold, which may be easier to select after these operations have been performed.

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1. Shrinking and expanding operations

"Shrinking" and "expanding" operations on two-valued digital pictures are useful for noise removal and segmentation [1, pp. 362-365 and 394-397]. Let Π be such a picture whose points all have value 0 or 1, and suppose that Π suffers from "salt-and-pepper noise" -- in other words, that Π contains regions consisting primarily of 0's with a sprinkling of isolated 1's and vice versa. We can remove isolated 1's by performing a "shrinking" operation, in which 1's are changed to 0's if they have 0's as neighbors, followed by an "expanding" operation in which 0's are changed to 1's if they have 1's as neighbors. This process shrinks large regions of 1's and then reexpands them to (essentially) their original state; but it annihilates isolated 1's, since they disappear under the shrinking and cannot be regenerated by the reexpansion. In fact, this process destroys any set of 1's that is no more than two pixels wide. Similarly, expanding followed by shrinking destroys isolated 0's (or sets of 0's that are at most two pixels wide).

More generally, if we use k repetitions of shrinking followed by k repetitions of expanding, we eliminate sets of 1's that are at most 2k pixels wide. If such a set is connected and has sufficiently large area (e.g., $\geq 10k^2$), it must be elongated, since its "length" (= area/width) is much greater than its width. Thus shrinking and expanding operations can be used to detect elongated parts of objects in a segmented digital picture. Another application of such oper-

ations is to the detection of clusters, or dense regions, in a picture composed of isolated 1's on a background of 0's (or vice versa). If we perform k repeated expansions, where k is at least half the distance between the 1's in a cluster, the cluster will "fuse" into a solid region; when we subsequently perform k repeated shrinks, this region will remain large (relative to k^2). On the other hand, 1's that do not belong to clusters will expand but not fuse with other 1's, so that when we shrink, they shrink back to single 1's.

2. Generalization to grayscale pictures

Shrinking and expanding operations are defined only for two-valued digital pictures. Suppose that one has a picture Π containing dark objects on a light background, but which suffers from (grayscale) "salt-and-pepper noise" -- in other words, a sprinkling of light points on the objects, and of dark points on the background. To apply shrinking and expanding to Π, we must first threshold Π so that dark points become 1's and light points become 0's. Under some circumstances, it may be undesirable to threshold Π at too early a stage in its processing, since this discards most of the information about the gray levels of the points of Π. Thus it would be desirable to define analogs of shrinking and expanding that can be applied directly to grayscale pictures.

In a two-valued picture consisting of 0's and 1's, shrinking the 1's is equivalent to computing the logical AND of each pixel with its neighbors, and expanding the 1's is equivalent to logically ORing each pixel with its neighbors. In the theory of fuzzy sets [2], the analogs of AND and OR are MIN and MAX, respectively. Thus a reasonable generalization of shrinking and expanding to grayscale pictures would be local MIN and MAX -- i.e., replacing the gray level of each pixel by the minimum (or maximum) of the gray levels of it and its neighbors. Clearly these operations do behave analogously to shrinking and expanding -- e.g., they should be effective in cleaning up grayscale salt-and-pepper noise. [Their use to detect elongated parts of objects, or clusters

of points, still requires some sort of thresholding, since these processes depend on detecting large connected components; but this thresholding can be performed at a later stage, when it may be easier to choose a good threshold.] Note also that if the only gray levels in a picture are 0 and 1, local MIN and MAX reduce to shrinking and expanding.

3. Local MIN and MAX commute with thresholding

In this section we show that if local MIN and MAX operations are applied to a picture Π, and Π is then thresholded (using any threshold t), the results are exactly the same as if Π were first thresholded at t, and shrinking and expanding operations were then applied to the thresholded Π. This implies that there is no need to threshold a picture in order to apply shrinking and expanding operations to it. Instead, we can apply the analogous set of local MIN and MAX operations, and thresholding can be deferred until afterwards (when it may be easier to choose a good threshold).

We can actually prove a more general result, namely that local MIN and MAX commute with any monotonic transformation of the grayscale. Let h be such a transformation, so that for all gray levels z_1 and z_2 we have $z_1 \le z_2$ implies $h(z_1) \le h(z_2)$. Then we can prove

<u>Proposition 1</u>. Local MIN and local MAX commute with h.

<u>Proof</u>: Since h is monotonic, the neighbor(s) for which the minimum (or maximum) value is taken on remain the same when h is applied.//

<u>Corollary 2</u>. Any sequence of local MIN and MAX operations commutes with h.//

<u>Corollary 3</u>. Any such sequence commutes with thresholding.

<u>Proof</u>: Thresholding is a monotonic grayscale transformation.//

These results imply

__Proposition 4__. Any sequence of local MIN and MAX operations followed by thresholding produces the same result as thresholding followed by the corresponding sequence of shrinking and expanding operations.

__Proof__: This follows from Corollary 3 and from the fact that, for a picture whose only gray levels are 0 and 1, local MIN is the same as shrinking and local MAX is the same as expanding.//

4. Examples

Figure 1 shows the results of repeatedly applying local MIN, and then repeatedly applying local MAX, to three noisy infrared (FLIR) images showing light objects on a dark background, for 1, 2, and 3 repetitions, using either 4 or 8 neighbors of each point to define the operations. It is seen that considerable noise cleaning is achieved. Note that there is also some smoothing of the object shape, especially for the larger numbers of repetitions; this is particularly noticeable for the smaller objects (Figs. 1b-c). Figure 2 shows analogous results for an artificial picture consisting of two noisy rectangles on a noise background; here again, substantial noise cleaning is obtained.

5. Concluding remarks

By Proposition 4, if we apply local MIN and MAX operations to a picture, the result can be thought of as encompassing the analogous shrinking and expanding results for all possible thresholdings of the picture. It may thus be preferable to use local MIN and MAX operations prior to thresholding, since this allows the choice of a threshold to be postponed. Under some circumstances, threshold selection may be easier after the operations have been performed; for example, if a picture contains dark objects on a light background together with grayscale salt-and-pepper noise, the noise may blur the valley on the picture's histogram between the peaks representing the object and background, so that it may be difficult to select a good threshold that separates these peaks (see [1, pp. 263-268]). Figure 3 shows the histograms of the pictures in Figure 1c; note that a narrow cleft develops in the histogram valley as a result of the local MIN/MAX processing.

Shrinking and expanding operations would be most easily implemented on a parallel array processor, and the same is true for local MIN and MAX operations. On a conventional computer, one pass through the picture is required for each iteration (or iterations can be combined by using a larger neighborhood of each pixel). In either case, the computational cost of local MIN and MAX operations is slightly higher, since they involve numerical comparisons rather than logical operations. However, this higher cost may be offset by the

potential advantages of local MIN and MAX in permitting the commitment to a particular threshold to be deferred.

APPENDIX A

SOME PROPERTIES OF ITERATED LOCAL MIN AND MAX

Let $\Pi^{(n)}$ denote the result of applying n iterations of local MAX to Π.

<u>Proposition A.1.</u> $\Pi^{(n)}(P)$ is the maximum of the values of Π within distance n of P.

<u>Note</u>: "Distance" in this proposition should be defined as follows: By a path of length m from P to Q is meant a sequence of points $P = P_0, P_1, \ldots, P_m = Q$ such that P_i is a neighbor of P_{i-1}, $1 \leq i \leq m$ (where "neighbor" means the same thing that it does in the definition of local MAX). We say that Q is within distance n of P if there exists a path of length n from P to Q. Readily, this notion of "distance" satisfies the definition of a <u>metric</u> on the set of points of Π: it is positive definite, symmetric, and satisfies the triangle inequality. Note also that the points at distance 1 from P are just the neighbors of P.

<u>Proof</u>: Let $D_n(P)$ (the "disk" of radius n centered at P) be the set of points within distance n of P. We first show that $D_n(P) = \bigcup_Q D_{n-1}(Q)$, where the union is taken over all Q at distance 1 from P (i.e., over all neighbors Q of P). Indeed, by the triangle inequality, any point in the union has distance $\leq$ n from P; and conversely, any point R at distance $\leq$ n from P is at distance $\leq$ n-1 from some neighbor Q of P, since a path from R to P must pass through some such Q.

We can now prove the proposition by induction on n. It is true by definition for $n = 1$; suppose it true for $n-1$, so that $\Pi^{(n-1)}(Q)$ is the maximum of values of Π within distance $n-1$ of Q, for all Q. Now $\Pi^{(n)} = (\Pi^{(n-1)})^{(1)}$ at P is the local MAX of the values of $\Pi^{(n-1)}$ in the neighborhood of P. By the first paragraph of the proof, this is just the maximum of the values of Π within distance n of P.//

Let $\Pi^{(-n)}$ denote the result of n iterations of local MIN applied to Π. Let $\bar{\Pi} = 1-\Pi$ (where we assume that the range of values of Π is $[0,1]$.

Proposition A.2. $\overline{\Pi^{(-n)}} = \bar{\Pi}^{(n)}$; $\overline{\Pi^{(n)}} = \bar{\Pi}^{(-n)}$

Proof: The local MAX M at a given point in $\bar{\Pi}$ is taken on at the same neighbor(s) as is the local MIN m at that point in Π; hence $M = 1-m$. The proposition follows immediately for $n = 1$; and by Proposition A.1, if we take "*local*" to mean "within distance n," the same proof holds for any n.//

Corollary. $\Pi^{(-n)} = \overline{\bar{\Pi}^{(n)}}$; $\Pi^{(n)} = \overline{\bar{\Pi}^{(-n)}}$.//

Proposition A.3. $(\Pi^{(n)})^{(-n)} \geq \Pi$; $(\Pi^{(-n)})^{(n)} \leq \Pi$.

Proof: For any Q within distance n of P, by Proposition A.1 we evidently have $\Pi^{(n)}(Q) \geq \Pi(P)$; hence the MIN (over all such Q) of the $\Pi^{(n)}(Q)$'s is also $\geq \Pi(P)$. The second part follows similarly.//

These propositions are analogous to well-known properties of expand/shrink operations [1]:

1) If we expand the set of 1's in a two-valued digital picture, n times, we obtain the set of points whose distances to the nearest 1 are $\leq$ n.

2) Expanding the 1's is the same as shrinking the 0's, and vice versa.

3) If we expand the 1's n times, and then shrink the result n times, we obtain a set that contains the original set of 1's. If we shrink the 1's n times, and then expand the result n times, we obtain a set that is contained in the original set of 1's.

APPENDIX B

RELATIONSHIP BETWEEN LOCAL MAX OR MIN AND FUZZY DISTANCE

Let S be the set of 1's in a two-valued digital picture, and for any point P, let d(P,S) be the distance from P to (the nearest point of) S, as defined in Appendix A. When we expand S, we add to it all points at distance 1 from it, and we decrease the distance from any $P \notin S$ to S by exactly 1. This appendix introduces the concept of distance between a point and a fuzzy set, and shows how performing local MAX decreases this distance by 1 (in a certain sense). We recall that a fuzzy set (defined on the points of the picture array) is a mapping μ of these points into the interval [0,1]; thus a picture Π, if we take its gray level range to be [0,1], defines a fuzzy set. If μ takes on only the values 0 and 1, it defines an ordinary set (the set of points mapped into 1).

Given a point P and a fuzzy set μ, we define a mapping δ of the positive x-axis into [0,1] by

$$\delta(x) \equiv \max_{Q \in D_x(P)} \mu(Q)$$

where $D_x(P)$ is the set of points within distance x of P. Thus $\delta(x)$ is the highest μ value at any point within distance x of P. The following properties of δ are immediate:

1) δ is monotonic nondecreasing, i.e., $x_1 \leq x_2$ implies $\delta(x_1) \leq \delta(x_2)$.
2) If μ takes on only the values 0 and 1, let S be the set of points it maps into 1; then $\delta(x) = 0$ for $x < d(P,S)$, and

$\delta(x) = 1$ for $x \geq d(P,S)$. Thus if μ defines an ordinary set S, δ is a step function whose step is at $d(P,S)$.

3) If μ and ν are two fuzzy sets that define the mappings δ_μ and δ_ν, respectively, then $\mu \leq \nu$ implies $\delta_\mu \leq \delta_\nu$.

We can now show that if we apply local MAX to μ, then for any point P, the mapping $\delta(x)$ is shifted one unit to the left, i.e., it becomes $\delta(x+1)$. Indeed, suppose that the new $\delta(x) = \alpha$; thus α is a local MAX at some point in $D_x(P)$, so that it is the original value at a neighbor of some such point, i.e., at some point of $D_{x+1}(P)$, which implies that $\alpha \leq \delta(x+1)$. Conversely, let $\delta(x+1) = \beta = \mu(Q)$, where $Q \in D_{x+1}(P)$; then Q has a neighbor in $D_x(P)$, so that when we apply local MAX, there is a point in $D_x(P)$ whose value $\geq \beta$, implying that the new $\delta(x) \geq \beta$.

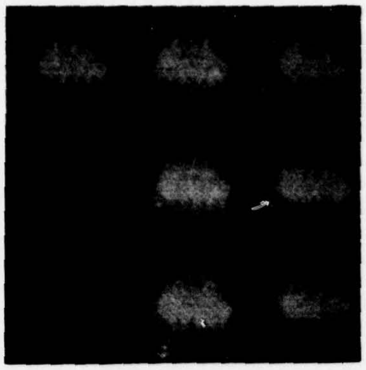
Note that the analogous result for local MIN (or for shrinking, in the non-fuzzy case) does not hold; for example, when we shrink a set S, we may increase the distance from $P \in S$ to S by much more than 1. On the other hand, shrinking S is the same as expanding its complement $\bar{S}$, and applying local MIN to μ is the same as applying local MAX to $1-\mu$ (see Appendix A); thus we have analogous results for shrinking and local MIN in terms of distance to the (fuzzy) complement.

References

1. A. Rosenfeld and A. C. Kak, Digital Picture Processing, Academic Press, N. Y., 1976.

2. L. A. Zadeh, Fuzzy sets, Information and Control 8, 1965, 338-353.

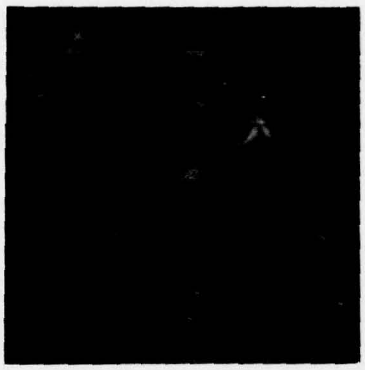
(a)



Key:

	4-nbr.	8-nbr.
Original	MIN·MAX	MIN·MAX
	$MIN^2 \cdot MAX^2$	$MIN^2 \cdot MAX^2$
	$MIN^3 . MAX^3$	$MIN^3 \cdot MAX^3$

(b)



(c)

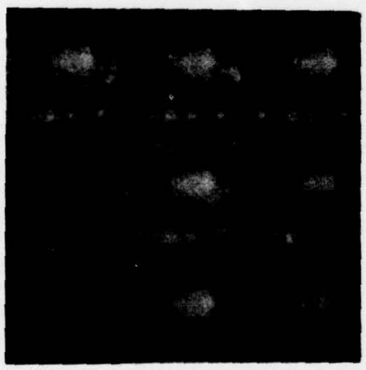


Figure 1. Results of applying repeated local MIN and repeated local MAX to three FLIR images. In each part, the upper-left picture is the original; the second column uses 4-neighbor local MINs followed by 4-neighbor local MAXes (1, 2, and 3 repetitions, in the first, second, and third rows); and the third column is analogous, using 8-neighbor operations (i.e., including the diagonal neighbors).

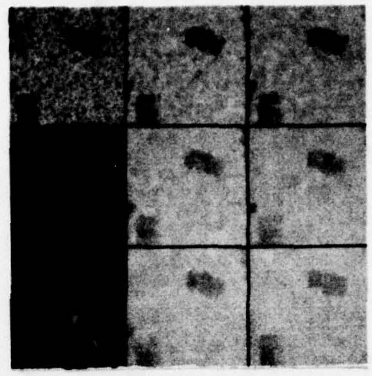


Figure 2. Analogous to Figure 1, using an artificial picture consisting of noisy rectangles on a noisy backgroud. (Mean gray levels 20 and 40, and standard deviation 5, on a 0-63 grayscale.)

Figure 3. Histograms of the pictures in Figure 1c. Note the development of a cleft at the bottom of the histogram valley.

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"Shrinking" and "expanding" operations on two-valued digital pictures are useful for noise removal, as well as for detecting "dense" regions and elongated parts of objects. For grayscale pictures, the analogs of shrinking and expanding are local MIN and MAX operations. These operations commute with thresholding; thus if they are applied to a grayscale picture, followed by thresholding, the result is the same as if the picture were first thresholded and shrinking and expanding were then performed. Applying MIN and MAX operations prior to thresholding

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ABSTRACT (cont'd)

thus makes it possible to defer the choice of a threshold, which may be easier to select after these operations have been performed.